Number Systems

Natural Numbers

How many ones are there in 642?

Natural Numbers

Aha!

642 is 600 + 40 + 2 in **BASE 10**

The **base** of a number determines the number of digits and the value of digit positions

(Why was base 10 chosen by humans?)

Positional Notation

Continuing with our example…

642 in base 10 *positional notation* is:

$$
\begin{aligned}
& 6 \times 10^2 = 6 \times 100 = 600 \\
+ & 4 \times 10^1 = 4 \times 10 = 40 \\
+ & 2 \times 10^0 = 2 \times 1 = 2 = 642 \text{ in base 10}
\end{aligned}
$$

This number is in base 10

The power indicates the position of the number

Positional Notation

What if 642 has the base of 13?

$$
\begin{array}{rcrcr}
6 \times 13^2 & = & 6 \times 169 & = & 1014 \\
+\ 4 \times 13^1 & = & 4 \times 13 & = & 52 \\
+\ 2 \times 13^0 & = & 2 \times 1 & = & 2 \\
 & & & = & 1068 \text{ in base } 10
\end{array}
$$

642 in base 13 is equivalent to 1068 in base 10

Octal

Decimal is base 10 and has 10 digits:
0,1,2,3,4,5,6,7,8,9

Octal is base 8 and has 8 digits:
0,1,2,3,4,5,6,7

For a number to exist in a given number system, the number system must include those digits.

For example:
The number 284 only exists in base 9 and higher.

Converting Octal to Decimal

What is the decimal equivalent of the octal number 642?

$$
\begin{array}{rcrcr}
6 \times 8^2 & = & 6 \times 64 & = & 384 \\
+\ 4 \times 8^1 & = & 4 \times 8 & = & 32 \\
+\ 2 \times 8^0 & = & 2 \times 1 & = & 2 \\
 & & & = & 418 \text{ in base 10}
\end{array}
$$

Bases Higher than 10

How are digits in bases higher than 10 represented?

Base 16:

0,1,2,3,4,5,6,7,8,9,A,B,C,D,E, and F

Converting Hexadecimal to Decimal

What is the decimal equivalent of the hexadecimal number DEF?

$$
\begin{array}{rcrcr}
D \times 16^2 & = & 13 \times 256 & = & 3328 \\
+\ E \times 16^1 & = & 14 \times 16 & = & 224 \\
+\ F \times 16^0 & = & 15 \times 1 & = & 15 \\
 & & & = & 3567 \text{ in base } 10
\end{array}
$$

Remember, base 16 is 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Converting Binary to Decimal

What is the decimal equivalent of the binary number 1101100?

$$
\begin{array}{rcrcr}
1 \times 2^6 & = & 1 \times 64 & = & 64 \\
+\ 1 \times 2^5 & = & 1 \times 32 & = & 32 \\
+\ 0 \times 2^4 & = & 0 \times 16 & = & 0 \\
+\ 1 \times 2^3 & = & 1 \times 8 & = & 8 \\
+\ 1 \times 2^2 & = & 1 \times 4 & = & 4 \\
+\ 0 \times 2^1 & = & 0 \times 2 & = & 0 \\
+\ 0 \times 2^0 & = & 0 \times 1 & = & 0 \\
 & & & = & 108 \text{ in base } 10
\end{array}
$$

Converting Decimal to Other Bases

<u>**Algorithm for converting base 10 to other bases:**</u>

(Note: stated differently in the book)

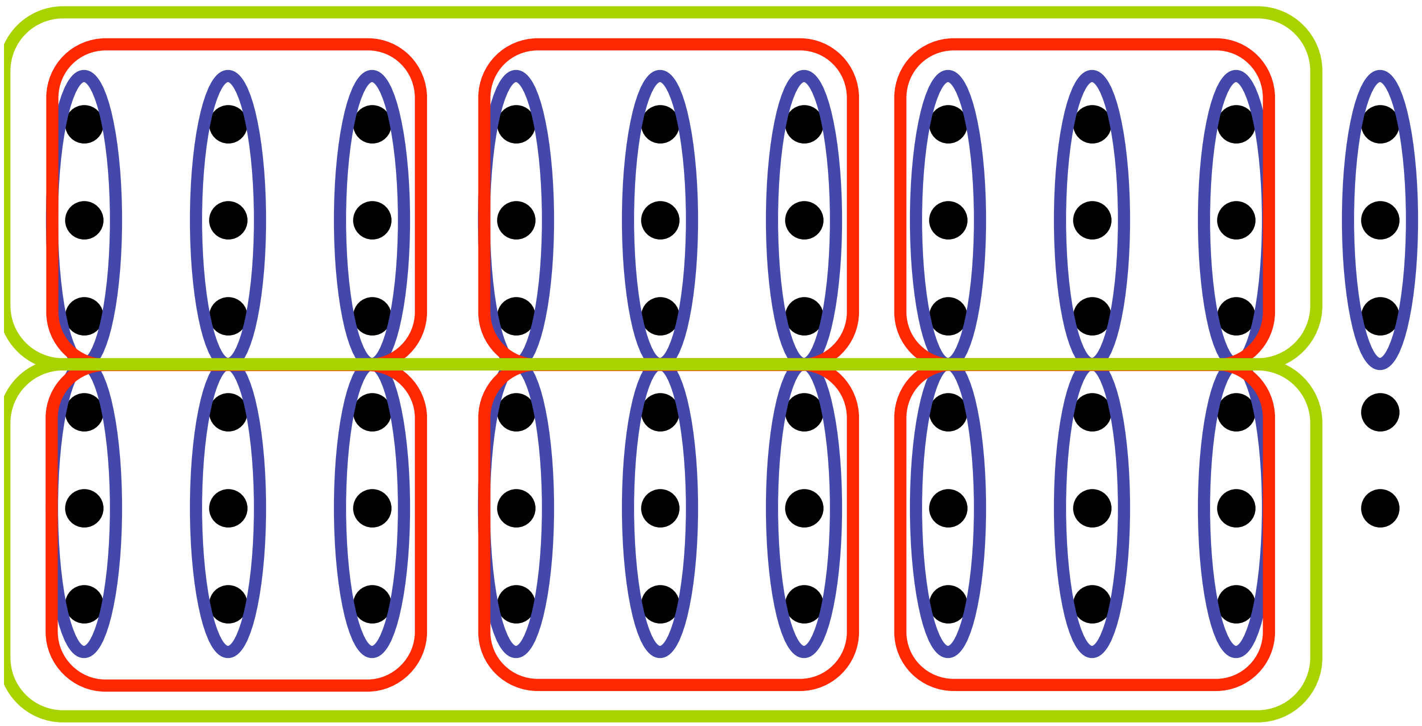
Repeat:

1. Divide the decimal number by the new base
2. Make the remainder the next digit
   to the left in the answer
3. Replace the decimal number with the quotient

Until the quotient is ***zero***

Why does this method work?

Let's convert 59 (base 10) to base 3:



19 R 2
3 | 59

6 R 1
3 | 19

2 R 0
3 | 6

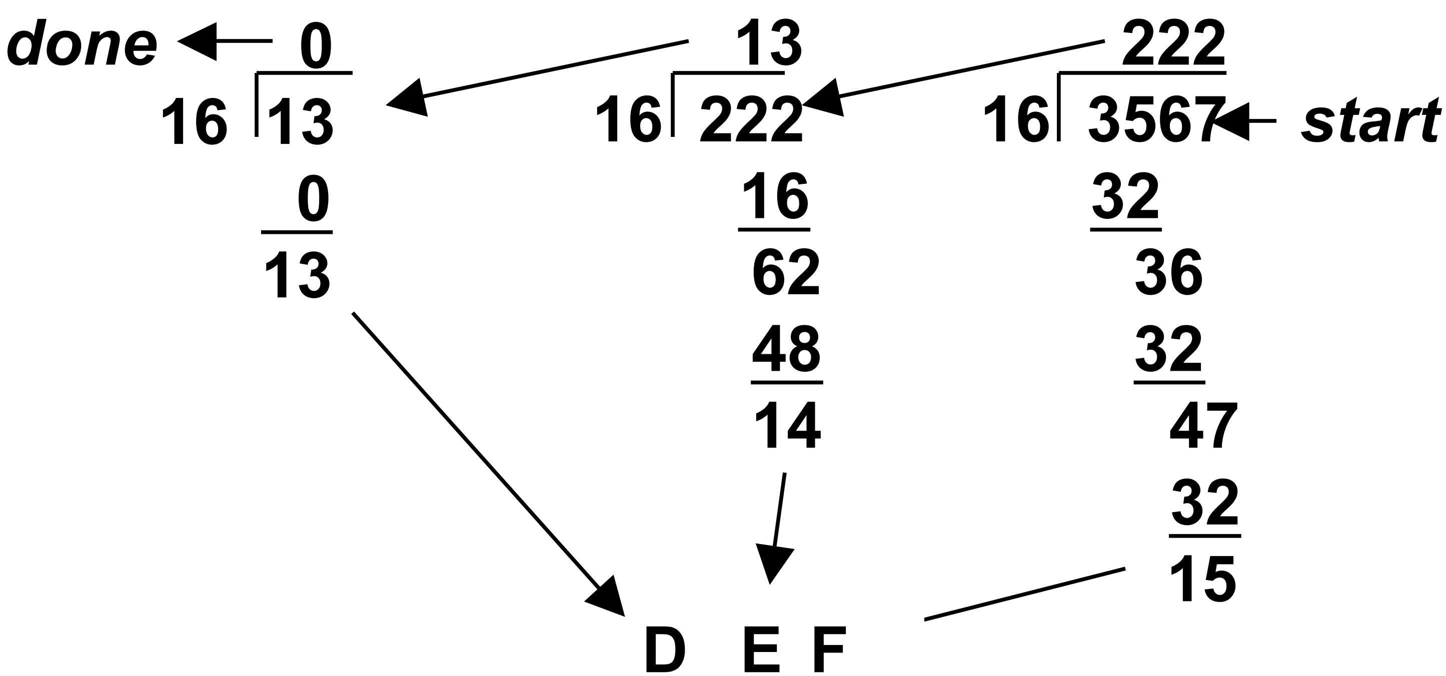
0 R 2
3 | 2

2 0 1 2

Converting Decimal to Hexadecimal

Try another Conversion:

3567 (base 10) is what number in base 16?



Converting Decimal to Other Bases

Try another Conversion:

The base 10 number 108

is what number in base 5?

Converting Binary to Octal

- Groups of Three (from right)
- Convert each group

10101011	**10**	**101**	**011**
	2	**5**	**3**

10101011 is **253** in base 8

Converting Binary to Hexadecimal

- Groups of Four (from right)
- Convert each group

10101011 **1010** **1011**

A **B**

10101011 is **AB** in base 16

Arithmetic in Decimal

Let's start with base 10:

$$\begin{array}{ccccccc}
 & 1 & & & 1 & 1 & \\
 & 3 & 5 & 7 & 2 & 5 & 7 \\
+ & & 6 & 2 & 5 & 4 & 5 \\
\hline
 & 4 & 1 & 9 & 8 & 0 & 2
\end{array}$$

Carry Values

Arithmetic in Other Bases

What if this is in base 8?

Carry Values

```
  1 1 1 1 1
  3 5 7 2 5 7
+   7 2 5 4 5
-------------
  4 5 2 0 2 4
```

5+7 = 8+4 = “14” (8)
4+5+1 = 8+2 = “12” (8)
5+2+1 = 8+0 = “10” (8)
2+7+1 = 8+2 = “12” (8)
7+5+1 = 8+5 = “15” (8)

Arithmetic in Binary

Remember: there are only 2 digits in binary: 0 and 1

Position is key, carry values are used:

Carry Values

```
 1   1  1 1 1 1
   1 0 1 0 1 1 1
 +1 0 0 1 0 1 1
 ---------------
 1 0 1 0  0 0 1  0
```

1+1 = 2 = 2 + 0 = 10 (2)
1+1+1 = 3 = 2 + 1 = 11 (2)
0+1+1 = 2 = 2 + 0 = 10 (2)
1+0+1 = 2 = 2 + 0 = 10 (2)
0+1+1 = 2 = 2 + 0 = 10 (2)

1+1 = 2 = 2 + 0 = 10 (2)

Subtracting Binary Numbers

Remember borrowing? Apply that concept here:

$$
\begin{array}{cccccccc}
 & & 1 & 2 & & & & \\
 & 0 & \cancel{2} & \cancel{0} & 2 & & & \\
 & \cancel{1} & \cancel{0} & \cancel{1} & \cancel{0} & 1 & 1 & 1 \\
- & & 1 & 1 & 1 & 0 & 1 & 1 \\
\hline
 & 0 & 0 & 1 & 1 & 1 & 0 & 0
\end{array}
$$